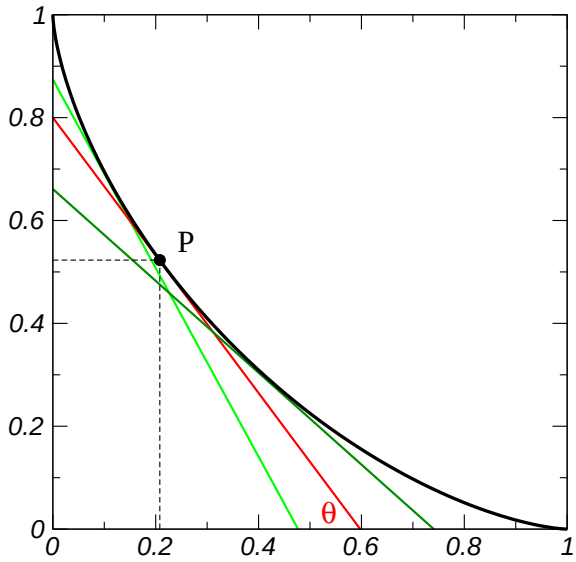
Ladder envelope

A ladder initially stands vertically against a wall. Its bottom end is given a sideways kick, causing the ladder to slide down. Assume that the bottom end is constrained to keep contact with the ground, and the top end is constrained to keep contact with the wall. Describe the envelope of the ladder's positions.

Answer of problem **Ladder envelope**

Envelope is the position of all points, for given slope θ of the ladder, that do not change under small change in θ. That is, if take point P and corresponding angle θ, a small change in ladder angle θ will leave this point intact.



In mathematical language this means that the ladder forms the tangent to the envelope at that point. We can parametrize the envelope as $[x(\theta), y(\theta)]$ and write this condition as

$$\frac{y'_\theta}{x'_\theta} = -\tan\theta\,,$$

and we also have the constraint on the length of the ladder, which we put to be unity:

$$\frac{x}{\cos\theta} + \frac{y}{\sin\theta} = 1$$

Solving these two equations togther, we obtain parametrical equation for the envelope

$$(x, y) = (\cos^3\theta, \sin^3\theta) \qquad 0 \le \theta \le \pi/2$$

or using the fundamental trigonometric identity,

$$x^{2/3} + y^{2/3} = 1$$