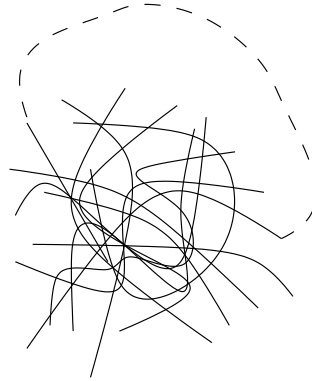


Spaghetti dinner

A bowl contains N spaghetti noodles. You reach into the bowl and grab two free ends at random and attach them. You do this N times until there are no free ends left. On average, how many loops are formed by this process?



Answer of problem **Spaghetti dinner**

Every time you connect two end you have one less noodle strand: you either made a loop or created a longer noodle.

On the first try you have N noodles and $2N$ ends. You pick an end and the probability that you pick up the other end of the same noodle and make a loop is 1 in $2N - 1$.

On the next try you have $N - 1$ noodles and $2N - 2$ ends. Probability that you get a loop this time is $1/(2N - 3)$. And so on, so the average number of noodle loops is

$$N_{loops} = \frac{1}{2N - 1} + \frac{1}{2N - 3} + \cdots + \frac{1}{3} + 1$$

For large number of noodles we can use approximation $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{N} = \ln N + \gamma$, with $\gamma \approx 0.5772$ being the Euler's constant.

$$N_{loops} = \sum_{n=1}^{2N} \frac{1}{n} - \frac{1}{2} \sum_{n=1}^{2N} \frac{1}{n} \approx \ln 2N + \gamma - \frac{1}{2}(\ln N + \gamma) = \frac{1}{2} \ln(4Ne^\gamma)$$

Bon appetit!