$$e^\pi \ ? \ \pi^e$$

On the xy plane determine region(s) where

$$x^y > y^x$$

and find which is greater, e^π or π^e?

Thanks to Recep Avci for suggesting this problem.

Answer of problem $e^\pi \ ? \ \pi^e$

Take log of both sides, and determine when

$$\frac{\ln(x)}{x} = \frac{\ln(y)}{y}$$

This equation has two solutions,

$$y = x$$

and another one that exists in $x > 1, \ y > 1$ region, and looks like a hyperbola, symmetric about $x = y$ line.

One can determine that

$$\boxed{e^\pi > \pi^e}$$