Year of the Double factorial

Show that the following fraction is an integer:

$$\frac{2012\,!! - 2011\,!!}{2013}$$

Answer of problem **Year of the Double factorial**

Represent $2012\,!! = 2 \cdot 4 \cdot 6 \cdot \ldots \cdot 2012$ as

$$2012\,!! = (2013-1)(2013-3)\ldots(2013-2011) = 2013\cdot(\textit{some integer}) + \underbrace{(-1)(-3)\ldots(-2011)}_{(-1)^{2012/2}\, 2011\,!!}$$

and the last term is cancelled by subtraction of $2011\,!!$. So we have a number divisible by 2013.

Second solution (Dana and several others):

2013 = 3 * 11 * 61

2011!! is a product of all odd integers .le. 2011... including 3, 11 and 61. Thus (2011!!)/2013 is an integer.

2012!! is a product of all even integers .le. 2012... including 6, 22 and 122. Thus (2012!!)/2013 is also an integer.

(2012!! - 2011!!)/2013 is therefore the difference between two integers and is itself an integer.

Anton's Note: in Dana's solution one realizes that we even can add the two numbers, and get an integer, which is not obvious from the first solution.

On the other hand, if the denominator is a prime number, the second approach might not be very useful (but then one can use so-called Wilson's theorem).