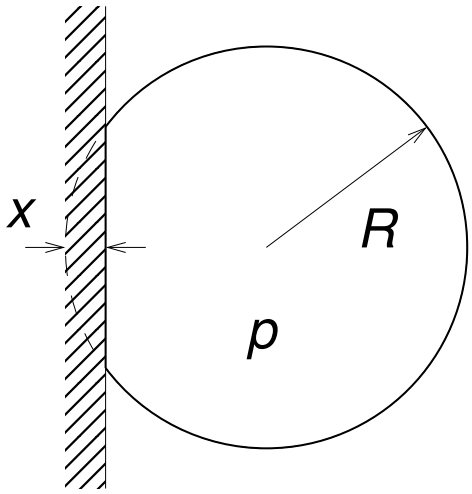
Hitting the woodwork

With the World Cup around the corner, we consider a soccer ball. During a weak strike of the ball against a wall, it gets deformed as shown in the figure, and we can assume that the deformation x is much less than the radius R, and that to a good approximation pressure p inside does not change during the interaction with the wall.

Neglecting elasticity of the ball's shell, estimate the time of collision with the wall. Find the numerical value of collision time, if mass of the ball is $m = 500$ g, $R = 12$ cm, $p = 2$ atm.



Answer of problem **Hitting the woodwork**

Answer

$$\boxed{\tau = \pi\sqrt{\frac{m}{2\pi R(p-p_0)}}}$$

where p_0 is the atmospheric pressure. For the given parameters of a soccer ball, this time is

$$\tau \approx 8 \cdot 10^{-3}\ sec$$

Solution:

The work done by the wall to change volume of the ball is

$$dW = pdV - p_0 dV = Fdx$$

where the volume of deformation is

$$V = \int_{R-x}^{R} \pi(R^2 - h^2)\, dh = \pi\left(Rx^2 - \frac{x^3}{3}\right)$$

and we have to also remember that this work includes the contribution from the atmosphere $-p_0 dV$.

So the force on the ball is

$$F = (p-p_0)\pi(2Rx - x^2) \approx 2\pi R(p-p_0)x$$

since $x \ll R$. This is a harmonic force that would result in oscillating motion with period

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{2\pi R(p-p_0)}}$$

The collision time is half the period (and independent of x).