Oscillator Integral

Find the value of the integral

$$I = \int_{x_1}^{x_2} \frac{dx}{\sqrt{(x - x_1)(x_2 - x)}}$$

by relating it to the motion of an ideal oscillator and just reading off the answer.

You can also just take the integral, of course, but this is too trivial.

Answer of problem **Oscillator Integral**

Half-period of an oscillator is

$$\frac{T}{2} = \frac{\pi}{\omega} = \int_{x_1}^{x_2} \frac{dx}{\sqrt{2(E - U(x))}}$$

where $U(x) = \omega^2 f(x)/2$ - is some quadratic function, which we can write

$$2E - U(x) = \omega^2(2E/\omega^2 - f(x)) = \omega^2(x - x_1)(x_2 - x)$$

where $x_{1,2}$ are the turning points. And so

$$I = \pi$$