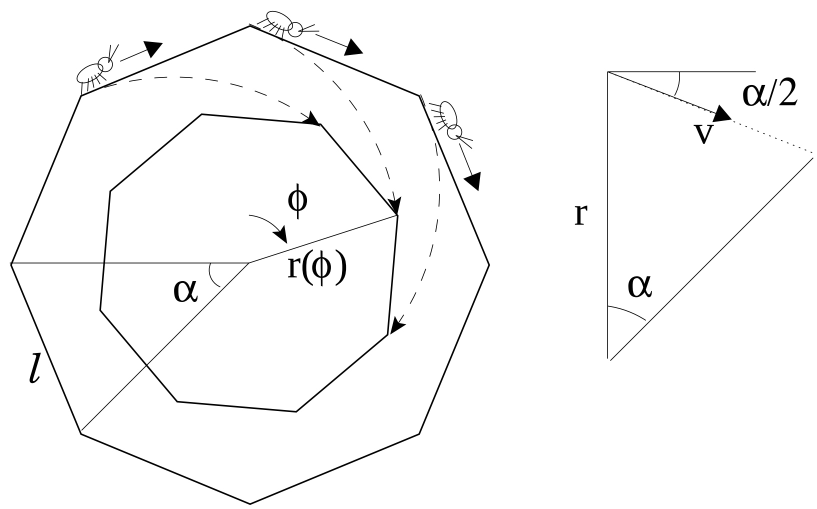
Bug vortex

N bugs are initially located at the vertices of a regular N-gon, whose sides have length ℓ. At a given moment, they all begin crawling with equal speeds in the clockwise direction, directly toward the adjacent bug. They continue to walk directly toward the adjacent bug, until they finally all meet at the center of the original N-gon. What is the total distance each bug travels? How many times does each bug spiral around the center?

Answer of problem **Bug vortex**



Due to symmetry, the bugs will always stay in the N-gon pattern, only twisted by angle ϕ, and each bug will be distance $r(\phi)$ from the origin.

At each moment we can write equation of motion for a bug in cylindrical coordinates,

$$\frac{dr}{dt} = -v\sin(\alpha/2) \qquad r\frac{d\phi}{dt} = v\cos(\alpha/2)$$

First equation gives the traveled distance:

$$L = \int v dt = -\int_R^0 \frac{dr}{\sin(\alpha/2)} = \frac{R}{\sin(\alpha/2)} = \frac{\ell}{2\sin^2(\alpha/2)} = \boxed{\frac{\ell}{1-\cos\alpha}} \qquad \alpha = \frac{2\pi}{N}$$

And the second gives the number of revolutions,

$$\Delta\phi = \int d\phi = \int \frac{d\phi}{dr}dr = -\int_R^0 \frac{dr}{r\tan(\alpha/2)} \to \boxed{\infty}$$

which is logarithmically divergent.