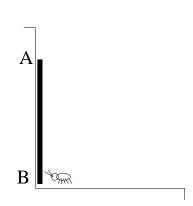
Problem of the Week

The bug climber

A stick of length L is placed vertically by the wall. At its lower end sits a bug. The end B of the stick starts moving to the right with speed v, and at the same moment the bug starts crawling along the stick with speed u relative to the stick. What is the maximal height above the floor that the bug reaches while it crawls along the stick? End A of the stick does not lose contact with the wall.



Answer of problem The bug climber

At time t, lower end of the stick will be distance vt from the wall, making angle $\cos \alpha = vt/L$ with the floor, and the bug progressed to a point ut from this end (along the hypotenuse). The height above the floor will be

$$h(t) = ut\sin\alpha = ut\sqrt{1 - \frac{v^2t^2}{L^2}}$$

Maximal height will be reached at time $t = \frac{L}{v\sqrt{2}}$, when the stick makes angle 45 deg with the floor, and it will be

$$h_{max} = \frac{Lu}{2v}$$

If the bug is fast, and it reaches the end of the stick (in time t = L/u) before the stick makes 45-angle, then the maximal height is

$$h_{max} = L\sqrt{1 - \frac{v^2}{u^2}}$$

