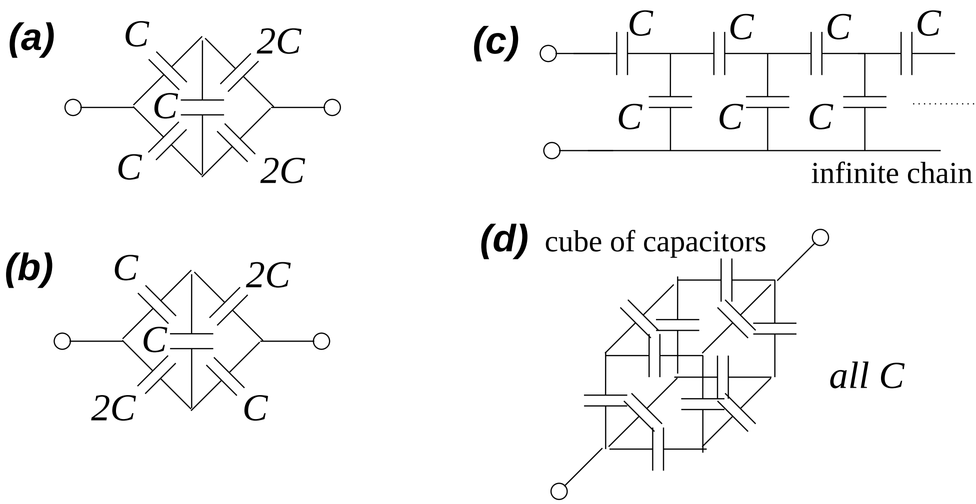
Capacitors



Use symmetries of the configurations to calculate their total capacitances.

Answer of problem **Capacitors**

(a) upper and lower branches are equivalent, so voltage across central capacitor is zero - throw it away.

$$C_a = 2\left(\frac{1}{C} + \frac{1}{2C}\right)^{-1} = \frac{4}{3}C$$

(b) From symmetry, voltage drops on outside C-capacitors are the same and we'll denote them U_1, and similarly for $2C$ capacitors, U_2, with condition $U_1 + U_2 = V$ - overall voltage drop. Voltage drop across middle one is $U_2 - U_1$. Write system of equations for charges on the capacitors, $q_1 = CU_1$, $q_2 = 2CU_2$ and $q_{center} = q_1 - q_2 = C(U_2 - U_1)$ and solve this system for $U_{1,2}$ and $q_{1,2}$. The overall capacitance is

$$C_b = \frac{q_1 + q_2}{V} = \frac{7}{5}C$$

(c) make a vertical cut after first vertical capacitor - resulting infinite chain is the same as the original

$$\frac{1}{C_c} = \frac{1}{C} + \frac{1}{C + C_c} \qquad \Rightarrow \qquad C_c = \frac{\sqrt{5} - 1}{2}C$$

(d) the points with the same voltages can be connected - this results in equivalent scheme: (3 parallel)-(6 parallel)-(3 parallel)

$$\frac{1}{C_d} = \frac{1}{3C} + \frac{1}{6C} + \frac{1}{3C} = \frac{5}{6C} \qquad \Rightarrow \qquad C_d = \frac{6}{5}C$$