

# Condensed Matter - HW 10 :: Superfluid

PHSX 545

## Problem 1

Using the phonon-roton model of  $^4\text{He}$  spectrum, find the quasiparticle entropy, heat capacity and density of the normal component at low temperature.

## Problem 2

(a) Find the energy  $E$  and the angular momentum  $L$  carried by a single vortex placed along the rotation axis of a long cylinder of radius  $R$ . Take the superfluid wave function in the form  $\psi(r, \phi) = \sqrt{n} e^{in\phi} \theta(r - r_c)$  with the uniform superfluid density  $\rho_s$  ( $n = 0, 1, 2, \dots$  and  $\theta(x)$  is the step-function). From this deduce the critical angular frequency  $\omega_c$  for nucleation of a single vortex (when  $E_{rot} = E - L\omega$  becomes negative).

(b) A neutron star of radius  $R = 10\text{km}$  rotates with frequency  $\Omega/2\pi = 1\text{ Hz}$ . Find the area density  $n_v$  of vortices in neutron superfluid, assuming the rotational motion of the star is due to vortices, their density is uniform, and the single-quantized vortices are along the rotation axis. Hint: calculate the circulation along a circle of radius  $r$  inside the star.