# Condensed Matter - HW 9 :: BEC & quasiparticles

PHSX 545

## Problem 1

Show that there is no BEC in two-dimensional ideal gas.

## Problem 2

Consider an excited configuaration of weakly interacting Bose gas at zero $T$, where two *quasiparticles* are present in a state with momentum $\mathbf{p} \neq 0$. Write down the quasiparticle wave function in Fock space, determine the occupation numbers of *particles*, and the number of particles in the condensate, compared to the ground state. Find the particle current carried by this state.

Hint: the single particle current operator at location $\mathbf{r}$ is

$$\mathbf{j}(\mathbf{r}) = \frac{1}{2m} \sum_i \left[\mathbf{p}_i \delta(\mathbf{r} - \mathbf{x}_i) + \delta(\mathbf{r} - \mathbf{x}_i)\mathbf{p}_i\right] \ ,$$

where summation is over all particles, $i$, $\mathbf{x}_i$ is particle $i$'s position, and its momentum operator $\mathbf{p}_i = \frac{\hbar}{i}\frac{\partial}{\partial \mathbf{x}_i}$. In terms of field creation and annihilation operators the current is

$$\mathbf{j}(\mathbf{r}) = \frac{1}{2m} \int d\mathbf{x} \hat{\Psi}^\dagger(\mathbf{x}) \left[\frac{\hbar}{i}\frac{\partial}{\partial \mathbf{x}}\delta(\mathbf{r} - \mathbf{x}) + \delta(\mathbf{r} - \mathbf{x})\frac{\hbar}{i}\frac{\partial}{\partial \mathbf{x}}\right] \hat{\Psi}(\mathbf{x}) \ ;$$

confirm that it coinsides in the form with the usual particle current of the Schrödinger equation, and write it in terms of particle operators $\hat{a}_\mathbf{p}, \hat{a}^\dagger_\mathbf{p}$ in plane wave basis.

## Problem 3

Consider a sudden change of the scattering length in Bose gas from $f_0$ to $F_0$. Both interactions are small. Within Bogoliubov theory, describe dynamics of the system.