Condensed Matter - HW 5 :: Zero Sound Attenuation

PHSX 545

Problem 1

Write the transport equation with collision term in τ approximation:

$$(\omega - \mathbf{q}\mathbf{v}_f)\nu_{\hat{\mathbf{p}}} - \mathbf{q}\mathbf{v}_f \int \frac{d\Omega_{\hat{p}'}}{4\pi} F^s(\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}')\nu_{\hat{\mathbf{p}}'} - \mathbf{q}\mathbf{v}_f\, U = -\frac{i}{\tau}[\nu_{\hat{\mathbf{p}}} - \nu_0 - \nu_1 P_1(\mathbf{q} \cdot \hat{\mathbf{p}})]$$

where we subtracted $\ell = 0$ and $\ell = 1$ terms in collision integral to satisfy the particle and momentum conservation laws.

(a) By projecting out different $P_\ell(\hat{\mathbf{p}} \cdot \hat{\mathbf{q}})$ harmonics derive general equation for ν_ℓ amplitudes directly from this equation, without dividing by $(\omega - \mathbf{q}\mathbf{v}_f)$ throughout (the latter we did in class, which resulted in $\Omega_{\ell\ell'}(s)$ functions). Hint: use the product property and one of the recursion relations ($xP_n(x) = \dots$) of Legendre polynomials.

(b) Assume $F^s(\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}')$ has non-zero $F^s_{\ell=0,1,2}$ terms only, and drop all others, $F^s_{\ell>2} = 0$. Write down equations for $\ell = 0, 1, 2, 3$ explicitly. Show that the $\ell = 0$ equation corresponds to particle number conservation, and try to show that $\ell = 1$ equation is momentum conservation (you might want to recall assignment two weeks ago).

(c) In the large $s = \omega/qv_f$ limit show that you can terminate the ν_ℓ series at $\ell = 2$. Set components $\ell > 2$ to zero and use equations for first three components ($\nu_{\ell=0,1,2}$) to find dispersion relation for sound wave s.

(d) Investigate the transition from first ($\omega\tau \ll 1$, expansion in $\omega\tau$) to zero ($\omega\tau \gg 1$, expansion in $1/\omega\tau$) sound, and explicitly determine temperature dependence of attenuation ($q = q' + iq''$) in the two limits.