Condensed Matter - HW 4 :: Impurity scattering

PHSX 545

Problem 1

The collision integral for elastic (energy-conserving) scattering of quasiparticles on impurities in Born approximation is given by

$$I_{imp}[n_{\mathbf{p}}] = \int d^3p' W(\mathbf{p}, \mathbf{p}')[-n_{\mathbf{p}}(1 - n_{\mathbf{p}'}) + n_{\mathbf{p}'}(1 - n_{\mathbf{p}})] = -\int d^3p' W(\mathbf{p}, \mathbf{p}')[n_{\mathbf{p}} - n_{\mathbf{p}'}]$$

where

$$W(\mathbf{p}, \mathbf{p}') = \frac{2\pi}{\hbar} |V(\mathbf{p} - \mathbf{p}')|^2 \delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}'})$$

is the Fermi Golden rule scattering amplitude, due to interaction V with impurities.

In relaxation time approximation we replace this integral by

$$I_{imp}[n_{\mathbf{p}}] = -\frac{\delta \bar{n}_{\mathbf{p}}}{\tau_{\mathbf{p}}} \qquad \text{with} \qquad \delta \bar{n}_{\mathbf{p}} = n_{\mathbf{p}}(\mathbf{r}) - \frac{1}{e^{\frac{\varepsilon_{\mathbf{p}}(\mathbf{r}) - \mu}{T(\mathbf{r})}} + 1}$$

being the deviation of distribution function from local equilibrium.

One can suggest different models for the scattering time $\tau_{\mathbf{p}}$. The simplest is just to take

$$\frac{1}{\tau(\varepsilon_{\mathbf{p}})} = \int d^3p' W(\mathbf{p}, \mathbf{p}') \tag{1}$$

which is momentum independent if $W(\mathbf{p} \cdot \mathbf{p}')$ is only function of the angle between scattered momenta. This is called the scattering life-time of a qusiparticle.

A better approximation for scattering time in transport problems is (self-consistently determined expression)

$$\frac{\delta \bar{n}_{\mathbf{p}}}{\tau_{\mathbf{p}}} = \int d^3p' W(\mathbf{p}, \mathbf{p}')[\delta \bar{n}_{\mathbf{p}} - \delta \bar{n}_{\mathbf{p}'}] \tag{2}$$

where it is implied that the found correction $\delta \bar{n}_{\mathbf{p}}$ is substituted back into RHS as $\delta \bar{n}_{\mathbf{p}'}$ to find self-consistent expression for $\tau_{\mathbf{p}}$.

(a) Using this expression for collision integral, derive the equation for deviation from equilibrium $\delta \bar{n}_{\mathbf{p}}(\mathbf{r})$ assuming that the temperature is a slow varying function of position $T(\mathbf{r})$.

(b) Find the scattering time $\tau_{\mathbf{p}}$ using result of (a). Write this scattering time explicitly. Hint: assume the deviation from equilibrium to be $\delta \bar{n}_{\mathbf{p}} = A(p)\ (\hat{\mathbf{p}} \cdot \boldsymbol{\nabla} T)$ where prefactor $A(p) = A(p')$ depends only on magnitude of p and you found it in (a). Use it in the collision integral to find τ; write $\hat{\mathbf{p}}' = \hat{\mathbf{p}}(\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}') + \hat{\mathbf{p}}'_{\perp}$ in the collision integral Eq.(2). and assume that the $\hat{\mathbf{p}}'_{\perp}$ integrates out to zero.

(c) Calculate the thermal conductivity κ using Boltzmann transport theory

$$\mathbf{q} = -\kappa \boldsymbol{\nabla} T \qquad \text{with definition} \quad \mathbf{q}(\mathbf{r}) = 2 \int \frac{d^3p}{(2\pi\hbar)^3} v_f \hat{\mathbf{p}} [\varepsilon_{\mathbf{p}}(\mathbf{r}) - \mu] \delta \bar{n}_{\mathbf{p}}(\mathbf{r})$$

Notice that the scattering time that enters κ is what you found in (b) and is different from Eq.(1). This time is called transport lifetime, and it reflects the fact that particles that forward-scatter $\mathbf{p} \to \mathbf{p}' \approx \mathbf{p}$ do not disturb the transport process very much.