Condensed Matter - HW3 :: Fermi Liquid Currents

PHSX 545

Problem 1

Various currents in Fermi liquid theory are given by the following expressions (spin-independent):

$$\mathbf{j} = \frac{1}{V}\sum_{\mathbf{p}}(\boldsymbol{\nabla}_{\mathbf{p}}\varepsilon^0_{\mathbf{p}})\,\delta\bar{n}_{\mathbf{p}} \qquad \text{particle current}$$

$$\Pi_{ij} = \frac{1}{V}\sum_{\mathbf{p}} p_i \frac{\partial \varepsilon^0_{\mathbf{p}}}{\partial p_j}\delta\bar{n}_{\mathbf{p}} \qquad \text{momentum current}$$

$$\mathbf{q} = \frac{1}{V}\sum_{\mathbf{p}} \varepsilon^0_{\mathbf{p}}(\boldsymbol{\nabla}_{\mathbf{p}}\varepsilon^0_{\mathbf{p}})\,\delta\bar{n}_{\mathbf{p}} \qquad \text{energy current (heat)}$$

where $\varepsilon^0_{\mathbf{p}}$ is energy in global equilibrium, and $\bar{n}_{\mathbf{k}}$ is deviation of distribution function from *local equilibrium*, and that includes interactions between quasiparticles.

(a) Calculate the particle current for a *single* excitation at momentum $\mathbf{p}$. Hint: in the absence of quasiparticle interactions this would have been just the group velocity $\mathbf{v}_p \approx v_f\hat{\mathbf{p}}$ of the particle. With quasiparticle interactions it will be a different velocity $\mathbf{u}$, that beside $\mathbf{v}_{\mathbf{p}}$ includes *backflow* currents from all other quasiparticles disturbed by the motion of the original one. Is it consistent with Galilean invariance principle (remember that we define co-moving reference frame by condition $\mathbf{j}' = 0$ - no particle current, in this case $\mathbf{p}' = 0$)?

(b) Calculate momentum and energy currents. For momentum current use symmetry arguments to identify Fermi liquid parameters $F^{s,a}_{\ell}$ that enter Π_{ij}. Theoretically minded part of the class are strongly encouraged to do this calculation fully.

The following expressions might help:

$$\int\frac{d\Omega_{\hat{k}}}{4\pi}\hat{k}_i = 0 \qquad \int\frac{d\Omega_{\hat{k}}}{4\pi}\hat{k}_i\hat{k}_j = \frac{1}{3}\delta_{ij} \qquad \int\frac{d\Omega_{\hat{k}}}{4\pi}\hat{k}_i\hat{k}_j\hat{k}_m = 0 \qquad \int\frac{d\Omega_{\hat{k}}}{4\pi}\hat{k}_i\hat{k}_j\hat{k}_m\hat{k}_n = \frac{1}{15}(\delta_{ij}\delta_{mn}+\delta_{im}\delta_{jn}+\delta_{jm}\delta_{in})$$