Condensed Matter - HW2 :: Legendre and Pauli

PHSX 545

Problem 1

(a) You don't need to derive this part, and may simply consult the math references. Decompose Legendre's polynomials in spherical harmonics

$$P_\ell(\cos\theta) = P_\ell(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}') = \sum_{m=-\ell}^{\ell} \ldots\ldots Y_{\ell m}(\hat{\mathbf{p}})\ldots\ldots$$

(b) Using the above and orthogonality of spherical harmonics calculate

$$\int \frac{d\Omega_{\hat{p}'}}{4\pi} P_{\ell_1}(\hat{\mathbf{p}}_1\cdot\hat{\mathbf{p}}')P_{\ell_2}(\hat{\mathbf{p}}_2\cdot\hat{\mathbf{p}}') = \cdots$$

where $d\Omega_{\hat{p}'} = \sin\theta_{\hat{p}'}\,d\theta_{\hat{p}'}\,d\phi_{\hat{p}'}$ is the solid angle integration over directions of $\hat{p}'$.

(c) Decompose f in Legendre polynomials and calculate:

$$\int \frac{d\Omega_{\hat{p}'}}{4\pi} f(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')\,\hat{\mathbf{p}}' = \cdots$$

Problem 2

(a) Write down anticommutator and commutator of Pauli matrices using δ_{ij} and ϵ_{ijk} tensors

$$[\sigma_i\,,\sigma_j]_+ = \ldots$$

$$[\sigma_i\,,\sigma_j]_- = \ldots$$

(b) Express a product of Pauli matrices in terms of (a) results

$$\sigma_i\sigma_j =$$

(c) Using (b) express in terms of scalar and vector products of $\mathbf{a}, \mathbf{b}$

$$(\mathbf{a}\boldsymbol{\sigma})(\mathbf{b}\boldsymbol{\sigma}) = \ldots$$

and find traces over spins

$$\mathrm{Tr}\,\{(\mathbf{a}\boldsymbol{\sigma})(\mathbf{b}\boldsymbol{\sigma})\} = \ldots$$

$$\mathrm{Tr}\,\{\boldsymbol{\sigma}(\mathbf{a}\boldsymbol{\sigma})(\mathbf{b}\boldsymbol{\sigma})\} = \ldots$$