Condensed Matter Theory: trial problems

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If you are interested in Condensed matter theory and superconductivity, you can use these problems to test your readiness, and figure out if you like the ideas in this field. Expected completion time to find solutions for these questions: 5-6 weeks. You can use literature. Some possible references:

- N. Ashcroft, D. Mermin 'Solid State Physics'
- A. Abrikosov 'Theory of metals'
- L. Landau, E. Lifshitz vols. IX, X
- M. Tinkham 'Introduction to superconductivity'
- *Phenomenological theory of unconventional superconductivity*, M. Sigrist, K. Ueda, Reviews of Modern Physics **63**, 239 (1991)

1. Find the angle-dependent density of states on the Fermi surface in a 2D metal with dispersion

$$\xi_{\mathbf{k}} = \xi(k_x, k_y) = \frac{1}{2m_x} k_x^2 + \frac{1}{2m_y} k_y^2 - \varepsilon_F$$

2. Calculate the electronic heat conductivity (using Boltzmann transport equation approach in relaxation time approximation) in a material with cylindrical Fermi surface and the elastic scattering rate of electrons

$$\Gamma_{\mathbf{k}\mathbf{k}'} = \frac{1}{\tau} \cos\Delta\phi \; \delta(\xi_{\mathbf{k}} - \xi_{\mathbf{k}'})$$

   where $\Delta\phi = \phi_{\mathbf{k}} - \phi_{\mathbf{k}'}$ is the scattering angle between points $\mathbf{k}$ and $\mathbf{k}'$ near the Fermi surface.

3. Write down the Ginzburg-Landau free energy for a single-component superconductor and find a solution for the order parameter near a surface with boundary condition $\Psi|_{surface} = 0$.

4. List some material examples of superconductors:
   (a) conventional
   (b) unconventional singlet
   (c) unconventional triplet
   (d) multiband

5. Find the elementary excitation energies in a non-unitary triplet superconductor, *i.e.* when $\mathbf{\Delta}_{\mathbf{k}} \times \mathbf{\Delta}^*_{\mathbf{k}} \neq 0$. For that find the eigenvalues of the matrix

$$\mathcal{H} = \begin{pmatrix} \xi_{\mathbf{k}} & \mathbf{\Delta}_{\mathbf{k}} \cdot \boldsymbol{\sigma} i\sigma_2 \\ -i\sigma_2 \boldsymbol{\sigma} \cdot \mathbf{\Delta}^*_{\mathbf{k}} & -\xi_{\mathbf{k}} \end{pmatrix}$$

   where $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices.

6. Find the temperature dependence of the order parameter $\Delta_{\mathbf{k}} \equiv \Delta(T)\mathcal{Y}(\hat{k})$ for a spherical Fermi surface, and isotropic $\mathcal{Y}(\hat{k}) = 1$, and anisotropic $\mathcal{Y}(\hat{k}) = \sin\theta$, superconductors. For that solve the self-consistency equation (numerically)

$$\Delta(T)\langle \mathcal{Y}(\hat{k})^2 \rangle \ln\frac{T}{T_c} = 2\pi T \sum_{m\geq 0} \left\langle \mathcal{Y}(\hat{k}) \left[\frac{\Delta_{\mathbf{k}}}{\sqrt{\varepsilon_m^2 + \Delta_{\mathbf{k}}^2}} - \frac{\Delta_{\mathbf{k}}}{|\varepsilon_m|} \right] \right\rangle$$

   where $\varepsilon_m = \pi T(2m+1)$ are the Matsubara energies, and the Fermi surface integral is angle integral over the sphere

$$\langle \dots \rangle = \int \frac{d\Omega_{\hat{k}}}{4\pi} \dots = \int \frac{d\phi \sin\theta d\theta}{4\pi} \dots$$

7. Find the analytic expression for the ratio $\Delta(T=0)/T_c$, for the two cases above.

8. The propagator of electron and hole excitations in a metal is $\hat{G}(z, \mathbf{k}) = (z\hat{\tau}_3 - \xi_{\mathbf{k}})^{-1}$ where $z = \varepsilon + i\delta$ is a complex energy, $\delta \neq 0$ and can be positive or negative; $\hat{\tau}_3 = \mathrm{diag}(1, -1)$ is the third Pauli matrix in particle-hole space; $\xi_{\mathbf{k}} = \mathbf{k}^2/2m - \mu$. Integrate to find the quasiclassical propagator defined by

$$\hat{g}(z) = \int_{-\infty}^{+\infty} d\xi_{\mathbf{k}} \, \hat{G}(z, \mathbf{k})$$